



Mathematical Model of the Drying Process of Wet Materials

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1. Statement of the problem

Heat treatment of wet materials is a technological process, accompanied by structural and mechanical changes of the dried substance. The research results in drying theory provide scientific basis to study this process intensification and selection of a rational method and optimal mode of heat treatment. The main technological task of heat treatment of materials is to develop combined methods of thermal action on the material with consideration of the main stages of its physico-chemical transformation. It is quite difficult to develop technology and appropriate methods for analytical description of heat treatment processes, involving successive physico-chemical transformations. Thermal swelling of the plastic raw mixture in production technologies of heat-insulating materials can serve as an example of this process. In this case, dehydration, gas formation, frame crystallization and drying processes are implemented. Therefore, the heat treatment method shall provide not only sufficient intensity, but also the best technological material properties.

Selection of rational technology of heat treatment of the material requires knowledge of its temperature field, as the dried product quality largely depends on the magnitude of temperature differences and duration of temperature exposure.

2. Identification of previously unsettled parts of the general problem

Detection of temperature fields and moisture content, when heated, is related to solution of a complex system of non-linear differential equations of heat and mass transfer with moving boundaries (Pavlenko 2018, Pavlenko 2020). However, the limited information on true values of moisture and heat transfer coefficients results only in a qualitative assessment of the processes, thus, complicating use of such solutions in engineering practice.

It is known (Nait-Ali et al. 2017, Dong et al. 2015, Maroulis et al. 2002), that experimentally determined heat transfer coefficients of wet bodies, when heated (thermal conductivity coefficient λ and temperature conductivity coefficient) are represented by effective values with consideration of heat and moisture transfer processes. Then, considering drying only as a thermal process, but with effective heat transfer coefficients with mass transfer, it is possible to obtain analytical dependences, convenient for engineering calculations, determining the temperature field and drying kinetics of wet materials.

3. Problem statement and methods of solution

Let us analyse the symmetrical heating process of wet raw material mixture, $2R$ thick, with initial moisture content \bar{U}_0 . Heat transfer from hot heat transfer agent to the material surface occurs according to the law of convective heat exchange at constant values of heat exchange coefficient α and heating medium temperature t_c .

Fig. 1 shows the experimental drying curve and temperature diagram of wet material ($2R = 0.16$ m) during heating process in an oven at constant temperature of heating air $t_c = 100^\circ\text{C}$ (wet-bulb temperature $t_m = 42^\circ\text{C}$). According to the scheme of sequential moisture removal from the material, developed by authors (Nuijten & Knut 2017, Tarnawski et al. 2002), in presented thermogram (Fig. 1) singular points (1-6) are marked, corresponding to a certain type of moisture binding with the body.

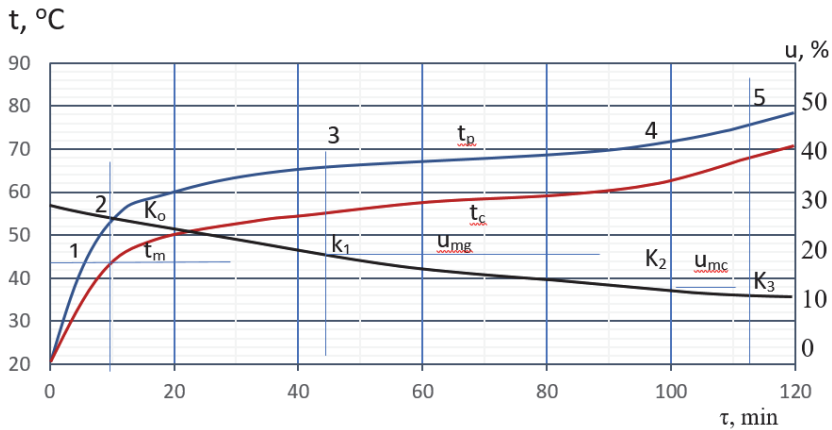


Fig. 1. Surface temperature $t_p(\tau)$, centre temperature $t_c(\tau)$ and moisture content $\bar{U}(\tau)$, constant temperature of heating medium $t_c = 100^\circ\text{C}$

Accordingly, the heating process can be conditionally divided into 6 stages:

- the first stage (0-1) runs at constant moisture content and ends when wet-bulb temperature of material surface is achieved ($t_p = t_m$),
- in the second stage (1-2) capillary moisture, contained in macropores, with negligible binding energy, is removed; at the stage end, temperature in the centre becomes equal to wet-bulb temperature ($t_c = t_m$), drying rate increases to the maximum,
- in the third stage (2-3) macropore moisture is removed from the material; medium-volume moisture content decreases to the maximum hygroscopic value $\bar{u}_{m.g}$ at the stage end, which corresponds to the first critical point k_1 on the drying curve; drying rate remains practicably unchanged,
- in the fourth stage (3-4), micropore capillary moisture is removed; medium-volume moisture content of the material decreases to the maximum adsorption $\bar{U}_{m.c}$, which corresponds to the second critical point k_2 on the drying curve,
- in the fifth (4-5) and sixth (5-6) stages, the polymolecular and monomolecular adsorption moisture is removed; medium-volume moisture content of the material varies from $\bar{U}_{m.c}$ to equilibrium \bar{U}_p .

To determine material temperature field in the first stage, with no moisture evaporation present, known solutions are used (Cherki et al. 2014), determining initial conditions for the next stage. The heating process in the second, third and subsequent stages occurs with deepening of evaporation surface of corresponding moisture type from outer surface into the material. Moisture evaporation inside material results in increased pressure of the vapour-air mixture, moving from front of evaporation to the outer body surface of the body and is removed into the environment. Each stage ends, when the surface boundary of phase transformation reaches the plate centre.

With mathematical statement of heat treatment problem, two zones are analysed at the second stage (Fig. 2): central zone 1, where moisture content is taken as constant and equal to initial ($U_1 = \bar{U}_0$), evaporation zone 2, where moisture content also remains unchanged and is equal to medium-volume moisture content of the material at the stage end ($U_2 = \bar{U}_{k0}$).

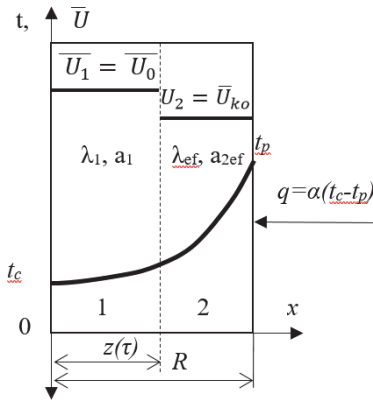


Fig. 2. For mathematical formulation of the problem

We consider that in zone 1 only moisture moves, that is, phase transformation criterion is $\varepsilon_1 = 0$, in evaporation zone 2 moisture moves as vapour ($\varepsilon_2 = 1$). The evaporation surface has a constant temperature equal to wet-bulb temperature t_m . The heat transfer coefficients in each zones are consider as constant.

The following expressions can be used to determine the values of effective thermal conductivity λ_{ef} and "net" thermal conductivity λ :

$$\lambda_{ef} = \frac{qR}{k_2 \Delta t_k}; \lambda_{ef} = \frac{(q - q_v)R}{k_2 \Delta t_k}, \tag{1}$$

where $q, q_v, \Delta t_k$ is a surface heat flow, a heat flow with evaporating moisture and temperature difference over the plate thickness at the end of this heating stage; k_2 is averaging factor of heat flows along the plate thickness, it depends on Vi criterion.

These assumptions allow to reduce the problem of heat and mass transfer (Dedic et al. 2003) to the problem of thermal conductivity with a moving boundary of phase transformation, its mathematical formulation includes differential equations of thermal conductivity for two zones:

$$\frac{\partial V_1}{\partial Fo} = \frac{\partial^2 V_1}{\partial x^2}, 0 \leq x \leq z(Fo) \tag{2}$$

$$\frac{\partial V_2}{\partial Fo} = f_a \frac{\partial^2 V_1}{\partial x^2}, z(Fo) \leq x \leq 1 \tag{3}$$

with boundary conditions

$$f_\lambda \left(\frac{\partial V_2}{\partial x} \right)_{x=1} = Bi(1 - V_2(1, Fo)) \tag{4}$$

$$V_1(z, Fo) = V_2(z, Fo) = V_M; \tag{5}$$

$$f\lambda \left(\frac{\partial V_2}{\partial x}\right)_{x=z(Fo)} - \left(\frac{\partial V_1}{\partial x}\right)_{x=z(Fo)} = \frac{d}{dFo} Ko(Fo) = \Delta Ko \frac{d}{dFo} z(Fo) \quad (6)$$

$$\left(\frac{\partial V_1}{\partial x}\right)_{x=0} = 0 \quad (7)$$

and initial conditions

$$V_1(x, 0) = V_0 + \Delta V_0 x^2; V_2(x, 0) = V_M; z(0) = 1; Ko(0) = Ko_n \quad (8)$$

Where $V = \frac{t(\chi, \tau)}{t_c}$ - relative temperature; $x = \frac{\chi}{R}$ - relative coordinate; $Fo = \frac{a_1 \tau}{R^2}$ - Fourier number; $Bi = \frac{a}{\lambda_1} R$ - Biot number; $Ko(Fo) = \frac{\bar{U}(Fo)r}{c_1 t_c}$ - Kosovich number; $Ki(Fo) = \frac{q(Fo)R}{\lambda_1 t_c}$ - Kirpichev criterion; $z(Fo) = \frac{z}{R}$ - relative coordinate of zone division; $\Delta Ko = \frac{r(\bar{U}_0 - \bar{U}_{k0})}{c_1 t_c}$; $V_M = \frac{t_M}{t_c}$; $V_0 = \frac{t_0}{t_c}$; $\Delta V = \frac{\Delta t_0}{t_c}$; $f\lambda = \frac{\lambda_{2ef}}{\lambda_1}$; $f_a = \frac{a_{2ef}}{a_1}$; $a = \frac{\lambda}{c\rho_0}$; $t(\chi, \tau)$ - temperature function; χ - coordinate; τ - time; λ_{ef}, a_{ef} - effective thermal conductivity and temperature conductivity coefficients; r - vaporization heat; C_l - specific heat of wet material, attributed to absolutely dry body mass; ρ_0 - absolutely dry body density; $\bar{U}(\tau)$ - medium-volume moisture content function; \bar{U}_{k0} - medium-volume moisture content at the end of the second heating stage; t_c - heating medium temperature; t_0 i Δt_0 - temperature of the centre and the temperature difference along plate thickness at the second stage beginning; index “1” refers to the central zone, index “2” - refers to evaporation zone.

The formulated problem (2)-(8) differs from classical Stefan problem in that the boundary condition on the outer plate surface is a time function, and the initial temperature distribution is expressed by a square parabola equation.

To solve the system of non-linear differential equations (2)-(8), the reduction and parametric perturbation (RPP) method is used (Lee et al. 2006), according to this procedure, general solutions of equations (2) i (3) look as follows:

$$V_1(x, Fo) = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} \cdot \frac{d^n}{dFo^n} \Psi(Fo) \quad (9)$$

$$V_2(x, Fo) = \sum_{n=0}^{\infty} \frac{(1-x)^{2n}}{(2n)! f_a^n} \cdot \frac{d^n}{dFo^n} \varphi(Fo) + \sum_{n=0}^{\infty} \frac{(1-x)^{2n}}{(2n+1)! f_a^n} \cdot \frac{d^n}{dFo^n} \delta(Fo) \quad (10)$$

where: $\Psi(Fo)$ and $\varphi(Fo)$ - temperature functions of the axis and plate outer surface; $\delta(Fo)$ - function of temperature gradient on the plate outer surface.

Solutions (9) and (10) meet the problem initial conditions, when: $\Psi(0)=V_0$;

$$\begin{aligned} \frac{d}{dFo} \Psi(0) &= 2\Delta V_0; \quad \frac{d^n}{dFo^n} \Psi(0)_{n \geq 2} = 0; \quad \varphi(0) = V_M; \quad \frac{d^n}{dFo^n} \varphi(0)_{n \geq 1} = 0; \\ \frac{d^n}{dFo^n} \delta(0) &= 0. \end{aligned} \quad (11)$$

Complying with solutions (9) and (10), boundary conditions of the problem, we obtain a system of non-linear standard differential equations, to which we introduce a conditional (small) parameter ξ :

$$-f_\lambda \delta(Fo) = Bi[1 - \varphi(Fo)] - \sum_{n=1}^{\infty} \frac{1}{(2n-1)!} \cdot \frac{d^n}{dFo^n} \psi(Fo); \quad (12)$$

$$\sum_{n=0}^{\infty} \frac{(1-\xi Z)}{(2n)! f_a^n} \cdot \frac{d^n}{dFo^n} \phi(Fo) + \sum_{n=0}^{\infty} \frac{(1-\xi Z)^{2n+1}}{(2n)! f_a^n} \cdot \frac{d^n}{dFo^n} \delta(Fo) = V_M; \quad (13)$$

$$\sum_{n=0}^{\infty} \frac{(1-\xi h)^{2n}}{(2n)!} \cdot \frac{d^n}{dFo^n} \psi(Fo) = V_M; \quad (14)$$

$$\begin{aligned} -f_\lambda \left\{ \sum_{n=1}^{\infty} \frac{(\xi h)^{2n-1}}{(2n)! f_a^n} \cdot \frac{d^n}{dFo^n} \psi(Fo) + \sum_{n=0}^{\infty} \frac{(\xi h)^{2n}}{(2n)! f_a^n} \cdot \frac{d^n}{dFo^n} \delta(Fo) \right\} &= \\ = \frac{d}{dFo} Ko(Fo) = N(Fo) = \Delta Ko \frac{d}{dFo} Z(Fo), \end{aligned} \quad (15)$$

where: $h(Fo) = 1 - z(Fo)$ – evaporation zone thickness function.

Required functions $\Psi(Fo)$, $\varphi(Fo)$, $\delta(Fo)$, $Ko(Fo)$, $N(Fo)$ and $z(Fo)$ are given in the form of the following expansions in a series by degrees of a small parameter ξ :

$$\left. \begin{aligned} \phi(Fo) &= \phi_o(Fo) + \xi \phi_1(Fo) + \xi^2 \phi_2(Fo) + \dots; \\ \psi(Fo) &= \psi_o(Fo) + \xi \psi_1(Fo) + \xi^2 \psi_2(Fo) + \dots; \\ \delta(Fo) &= \delta_o(Fo) + \xi \delta_1(Fo) + \xi^2 \delta_2(Fo) + \dots; \\ Ko(Fo) &= Ko_o(Fo) + \xi Ko_1(Fo) + \xi^2 Ko_2(Fo) + \dots; \\ N(Fo) &= N_o(Fo) + \xi N_1(Fo) + \xi^2 N_2(Fo) + \dots; \\ Z(Fo) &= Z_o(Fo) + \xi Z_1(Fo) + \xi^2 Z_2(Fo) + \dots; \\ h(Fo) &= h_o(Fo) + \xi h_1(Fo) + \xi^2 h_2(Fo) + \dots; \end{aligned} \right\} \quad (16)$$

By inserting series (16) in system of equations (12)-(15) and comparing coefficients at the same degrees of parameter ξ , we calculate a sequence of linear differential equations that determine the required functions.

Zero approximation (generating system of equations, determining temperature functions $\Psi_0(Fo)$, $\phi_0(Fo)$, $\delta_0(Fo)$ and medium-volume moisture content function $Ko_0(Fo)$, moisture removal rate $N_0(Fo)$, phase transformation boundary $z_0(Fo)$ in the absence of perturbations).

$$-f_\lambda \delta_o(Fo) = Bi[1 - \phi_o(Fo)] - \sum_{n=1}^{\infty} \frac{1}{(2n-1)!} \cdot \frac{d^n}{dFo^n} \psi_o(Fo); \tag{17}$$

$$\sum_{n=0}^{\infty} \frac{1}{(2n)! f_a^n} \cdot \frac{d^n}{dFo^n} \phi_o(Fo) + \sum_{n=0}^{\infty} \frac{1}{(2n+1)! f_a^n} \cdot \frac{d^n}{dFo^n} \delta_o(Fo) = V_m; \tag{18}$$

$$\sum_{n=0}^{\infty} \frac{1}{(2n)!} \cdot \frac{d^n}{dFo^n} \psi_o(Fo) = V_m; \tag{19}$$

$$f_\lambda \delta_o(Fo) = \frac{d}{dFo} Ko_o(Fo) = N_o(Fo) = \Delta Ko \frac{d}{dFo} Z_o(Fo). \tag{20}$$

The first approximation (a system of equations, determining the first complement to functions found in the zero approximation)

$$f_\lambda \delta_1(Fo) = Bi \phi_1(Fo) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)!} \cdot \frac{d^n}{dFo^n} \phi_1(Fo); \tag{21}$$

$$\sum_{n=0}^{\infty} \frac{1}{(2n)! f_a^n} \cdot \frac{d^n}{dFo^n} \phi_1(Fo) + \sum_{n=0}^{\infty} \frac{1}{(2n+1)! f_a^n} \cdot \frac{d^n}{dFo^n} \delta_1(Fo) - Z_o(Fo) \cdot \left\{ \sum_{n=1}^{\infty} \frac{1}{(2n-1)! f_a^n} \cdot \frac{d^n}{dFo^n} \phi_0(Fo) + \sum_{n=0}^{\infty} \frac{1}{(2n)! f_a^n} \cdot \frac{d^n}{dFo^n} \delta_o(Fo) \right\} = 0; \tag{22}$$

$$\sum_{n=0}^{\infty} \frac{1}{(2n)!} \cdot \frac{d^n}{dFo^n} \phi_1(Fo) - h_o(Fo) \cdot \sum_{n=1}^{\infty} \frac{1}{(2n-1)!} \cdot \frac{d^n}{dFo^n} \psi_o(Fo) = 0; \tag{23}$$

$$\frac{f_\lambda}{f_a} h_o(Fo) \frac{d}{dFo} \phi_o(Fo) + f_\lambda \delta_1(Fo) = \frac{d}{dFo} Ko(Fo) = N_1(Fo) = \Delta Ko \frac{d}{dFo} Z_1(Fo). \tag{24}$$

Similarly, the following complements to the main solution are formed.

For practical research of wet material heating processes, the first approximation provides sufficient accuracy.

Therefore, application of RPP method allows transforming the initial non-linear problem of thermal conductivity into a sequence of ordinary linear differential equations.

To solve obtained equations, Laplace's method of integral transformations is used. The calculated analytical dependences, determining temperature

field and drying kinetics of the wet plate are explicit functions, quite simply used in calculations.

Expressions for surface and plate axis temperature functions have the following form:

$$V_2(1, Fo) = V_m + \int_0^{Fo} \Phi_1(Fo - fo) \theta(fo) dfo + \phi_1(Fo); \quad (25)$$

$$V_1(0, Fo) = V_o + 2\Delta V_o \cdot G_1(Fo) + \psi_1(Fo), \quad (26)$$

where: $\varphi_1(Fo)$ and $\Psi(Fo)$ – complements to corresponding zero approximation functions.

$$\varphi_1(Fo) = \int_0^{Fo} \Phi_3(Fo - fo) M_1(fo) dfo; \quad (27)$$

$$\psi_1(Fo) = \int_0^{Fo} G_2(Fo - fo) M_2(fo) dfo \quad (28)$$

Function of evaporation surface coordinate

$$Z(Fo) = 1 - \frac{Bi}{\Delta Ko} \int_0^{Fo} \Phi_2(Fo - fo) \theta(fo) dfo + Z_1(Fo), \quad (29)$$

where: $Z_1(Fo) = \frac{1}{\Delta Ko} Ko_1(Fo)$ (30)

Function of medium-volume moisture content of the material

$$Ko(Fo) = Ko_n - Bi \int_0^{Fo} \Phi_2(Fo - fo) \theta(fo) dfo + Ko_1(Fo), \quad (31)$$

where:

$$Ko_1(Fo) = \int_0^{Fo} N_1(fo) dfo. \quad (32)$$

Function of moisture removal rate from material:

$$N(Fo) = Bi \int_0^{Fo} \Phi_3(Fo - fo) \theta(fo) dfo + N_1(Fo), \quad (33)$$

where:

$$N_1(Fo) = \frac{f_\lambda}{f_a} \cdot \frac{d}{dfo} \phi_o(Fo) [1 - Z_o(Fo)] + Bi \cdot \phi_1(Fo). \quad (34)$$

The following functions are used in computed expressions:

$$\Phi_1(Fo) = f_a \sum_{n=1}^{\infty} A_n \mu_n^2 \exp(-f_a \mu_n^2 Fo); \tag{35}$$

$$\Phi_2(Fo) = \frac{1}{1 + Bi_1} + \sum_{n=1}^{\infty} A_n \cdot \exp(-f_a \mu_n^2 Fo); \tag{36}$$

$$\Phi_3(Fo) = -f_a \sum_{n=1}^{\infty} \frac{A_n \mu_n^2}{\cos \mu_n} \exp(-f_a \mu_n^2 Fo); \tag{37}$$

where:

$$A_n = \frac{2Bi_1}{\mu_n^2 + Bi_1 + Bi_1^2}; \quad Bi_1 = \frac{Bi}{f_\lambda};$$

μ_n – equation roots: $\text{tg} \mu = -\mu / Bi_1$;

$$G_1(Fo) = \frac{1}{2} - \sum_{k=1}^{\infty} (-1)^{k+1} \frac{2}{\mu_k^3} \exp(-\mu_k^2 Fo); \tag{38}$$

where:

$$\mu_k = 0.5(2k-1)\pi;$$

$$G_2(Fo) = 1 - \sum_{n=1}^{\infty} \frac{2}{\mu_k^2} \exp(-\mu_k^2 Fo); \tag{39}$$

$$\theta(Fo) = 1 - V_m - \frac{2\Delta V_o}{Bi} [1 - G_2(Fo)]; \tag{40}$$

$$M_1(Fo) = -Z_o(Fo) \cdot Bi_1 \int_0^{Fo} \Phi_3(Fo - fo) \theta(fo) dfo; \tag{41}$$

$$M_2(Fo) = 2\Delta V_o [1 - Z_o(Fo)] [1 - G_2(Fo)]. \tag{42}$$

To determine the temperature field and drying kinetics in the third and subsequent stages of heating, the same analytical dependences are used, but with new initial conditions, obtained at previous stages.

4. Results and discussion

Calculation results of the heating process of a wet raw material block, $2R$ thick = 0.016 m at constant temperature of the heating medium (air) are shown in Fig. 3.

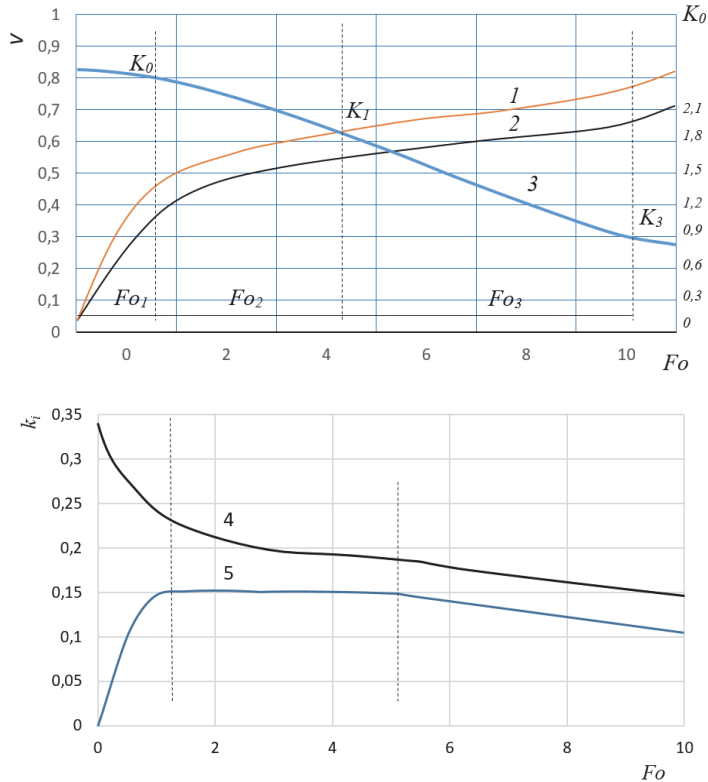


Fig. 3. Temperature and thermal diagrams: 1, 2 – relative temperature of a block surface and centre; 3 – dimensionless function of moisture content $K_0(Fo)$; 4 – surface heat flow $K_i(Fo)$; 5 – dimensionless rate of moisture removal from the material $N(Fo)$

Calculations were performed for the following conditions:

$$\bar{U}_0 = 0.37 \text{ kg/kg}; t_c=100^\circ\text{C}; t_M=42^\circ\text{C}; \alpha=15,7 \text{ W}/(\text{m}^2\text{K}); \rho_o = 530 \text{ kg}/\text{m}^3;$$

$$\lambda_1=0.2 \text{ W}/(\text{mK}); \lambda_{ef}=0.25 \text{ W}/(\text{mK}); a_1=0.1 \cdot 10^{-6} \text{ m}^2/\text{s}; a_{ef}=0.12 \cdot 10^{-6} \text{ m}^2/\text{s}.$$

Calculated data agree quite well with those found by experiment (Fig. 1), obtained by heating the samples in the oven under the same conditions.

5. Conclusions

Based on the studies of temperature field and moisture evaporation kinetics, the entire heating process can be divided into six stages according to the scheme of sequential removal of moisture from the material. The heating process in each stage runs with deepening of evaporation surface of corresponding moisture type from outer surface into the material. Each stage of symmetrical heating ends, when the surface boundary of phase transformation reaches the plate centre. Experimental data show that temperature distribution over material thickness at the end of each stage is parabolic. Therefore, using the formulas for regular heating mode, it is possible to determine effective heat transfer coefficients, considering heat and moisture transfer.

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Abstract

The problem of heat treatment of wet materials contains the question of the heat and mass inside the body transfer (an internal problem) and in the boundary layer at the interface between phases (an external problem). The amount of removable moisture depends on the degree of each of these processes development. When heated, the moisture content on the surface decreases, creating a concentration difference across the body. Therefore, a flow of moisture occurs in the body from deep layers to the surface, towards which the flow of heat is directed. Thus, when wet materials are heated, complex processes of moisture and heat exchange occur, mutually affecting the enthalpy and moisture content of both the heated material and the environment.

The features of mathematical model construction of heating and drying of wet materials process are considered in the article. The drying process is defined as a thermal process with effective heat transfer coefficients with consideration of mass transfer. It makes it possible to obtain analytical dependencies that are convenient for engineering calculations, with which you can determine the temperature field and evaluate the kinetics of wet materials drying.

Keywords:

thermal insulation materials, mathematical modelling, heat treatment, thermal conductivity, thermal processes

Model matematyczny procesu suszenia materiałów wilgotnych**Streszczenie**

Problem obróbki cieplnej wilgotnych materiałów obejmuje zagadnienia transferu ciepła i masy wewnątrz komponentu (problem wewnętrzny) i w warstwie granicznej z przemianą fazową (problem zewnętrzny). Ilość usuwanej wilgoci zależy od stopnia rozwoju każdego z tych procesów. Po podgrzaniu zawartość wilgoci na powierzchni zmniejsza się, tworząc różnicę koncentracji w całym materiale. Dlatego w materiale występuje przepływ wilgoci z głębokich warstw na powierzchnię, na którą skierowany jest przepływ ciepła. Oznacza to, że gdy ogrzewane są wilgotne materiały, zachodzą złożone procesy wymiany wilgoci i ciepła, wpływając wzajemnie na entalpię i zawartość wilgoci zarówno ogrzewanego materiału, jak i środowiska.

W artykule omówiono cechy budowy modelu matematycznego procesu ogrzewania i suszenia materiałów zawilgoconych. Proces suszenia definiuje się jako proces termiczny o efektywnych współczynnikach przenikania ciepła z uwzględnieniem transferu masy. Umożliwia uzyskanie zależności analitycznych dogodnych do obliczeń inżynierskich, za pomocą których można określić pole temperatury i ocenić kinetykę suszenia wilgotnych materiałów.

Słowa kluczowe:

materiały termoizolacyjne, modelowanie matematyczne, obróbka cieplna, przewodnictwo cieplne, procesy termiczne.